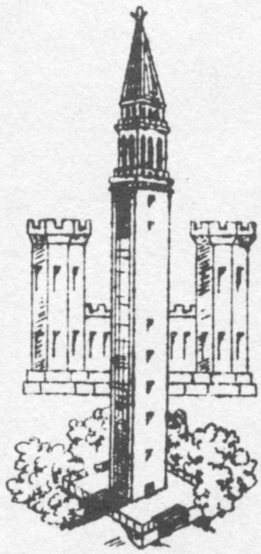
TRANSPORT OF SEDIMENT MIXTURES WITH LARGE RANGES OF GRAIN SIZES

By

H. A. Einstein and Ning Chien



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THE MISSOURI RIVER DIVISION
CORPS OF ENGINEERS
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CORPS OF ENGINEERS SEDIMENT STUDIES PROGRAM

FOR MISSOURI RIVER BASIN

MISSOURI RIVER DIVISION

FORT PECK DISTRICT	OMAHA DISTRICT
GARRISON DISTRICT	KANSAS CITY DISTRICT

The Corps of Engineers Missouri River Basin sediment studies program was established for the development of practical sediment engineering for rational evaluation, regulation, and utilization of fluvial sediment phenomena. It was implemented as a comprehensive, basin-wide program for coordination of studies of sediment problems in the overall basin program for flood control and allied purposes as well as for continuity and perspective in the planning and design of individual projects. The program includes both investigations for the development of sediment transport theory and observations of existent and occurring phenomena for the purpose of developing the applications of theory to practical problems, developing empirical relationships, and providing aids to judgment.

The program has been conducted during the tenures of and supported by the following Division Engineers:

Lieutenant General Lewis A. Pick
Major General Samuel Sturgis
Brigadier General Don G. Shingler
Brigadier General W. E. Potter

Mr. F. B. Slichter was Chief of the Engineering Division from the inception of the program until April 1949. Mr. W. E. Johnson has been Chief of the Engineering Division since that time. The program was formulated and organized by Mr. R. J. Pafford, Jr., Chief, Planning and Reports Branch. Planning and execution is under the immediate direction of D. C. Bondurant with technical advice and assistance provided by an Advisory Board consisting of:

Mr. G. A. Hathaway	Mr. T. H. Means
Dr. H. A. Einstein	Dr. L. G. Straub
Mr. E. W. Lane	Dr. V. A. Vanoni

TRANSPORT OF SEDIMENT MIXTURES WITH LARGE RANGES OF GRAIN SIZES

by

H. A. Einstein & Ning Chien

Introduction

The movement of sediment along the bed of alluvial rivers has long been one of the most perplexing and challenging problems confronting the hydraulic engineer. In the past few decades, because of the rapid expansion of activity in both engineering and geology, the importance of this problem has been gravely felt and great stimulus has been given to the search for a general solution. Since the classic experiment of Gilbert in 1914, numerous laboratory studies of sediment transportation have been conducted throughout the United States and the continent of Europe. Many sediment transportation formulas have been derived from these studies.

An examination of these studies reveals that in most of the cases uniform particles are used in conducting the experiments. The formulas thus derived can be directly applied only to sediment of uniform size, although some authors have proposed to extend their application to the case of mixtures by use of a summation process. This process virtually neglects the interaction between particles of different sizes in the mixture, and its applicability is rather doubtful. In a few of the cases where non-uniform materials are employed in the experiments, the effects of mixture are all included in one single factor. Kramer[(1)*] called this factor "uniformity modulus" which is more or less an indication of the average size spread in a mixture. Trask[(2)] proposed to measure this size spread in a mixture by a sorting coefficient of the following form:

$$S_o = \text{sorting coefficient} = (D_{25}/D_{75})^{1/2} \tag{1}$$

* Numerals in subscripts refer to corresponding items in Reference.

where D_{25} and D_{75} are the sieve sizes of the grains of which 25% and 75% by weight are finer respectively. For uniform material this coefficient is unity. Again it is not only questionable that such a complicated system as the mutual interference between bed particles of different sizes can be well defined by one factor alone, but this factor also fails to describe how this mutual interference takes place.

In 1950 a complete theory was presented[3] which will permit the calculation of the equilibrium rate at which various discharges will transport the different grain sizes of the bed material in a given channel. It was first concluded on the basis of special experiments that a given particle size moves in a series of steps of a constant average length, and that it is periodically deposited in the bed after performing such a step. The number of particles deposited per unit time in the unit of bed area may be expressed in terms of the rate of transport and the size and weight of the particle. The rate at which sediment particles of a certain size are eroded from the bed is proportional to the number of particles in the surface of the bed area and to the probability that such a particle in the bed surface is eroded during the unit time. This probability may be expressed also as the probability that the ratio of dynamic lift on the particle to the weight of the particle under water will be larger than unity. The equilibrium rate of bed material transportation is then obtained by equating the number of particles deposited on and eroded from the unit bed area per unit time; and this leads to the final bed load equation as follows:

$$1-\frac{1}{\sqrt{\pi}}\int_{-B_*\Psi_*-\frac{1}{\eta_o}}^{B_*\Psi_*-\frac{1}{\eta_o}} e^{-t^2}\,dt = \frac{A_*\Phi_*}{1-A_*\Phi_*} \qquad (2)$$

in which η_o, A_*, and B_* are universal constants. The equation is represented graphically by a single curve between the flow intensity ψ_* and the intensity of bed load transport ϕ_*, in which

$$\phi_* = \frac{i_B}{i_b}\phi = \frac{i_B}{i_b}\frac{q_B}{\rho_s g}\left(\frac{\rho_f}{\rho_s-\rho_f}\right)^{1/2}\left(\frac{1}{gD^3}\right)^{1/2} \tag{3}$$

$$\psi_* = \xi Y\left(\frac{\beta}{\beta_x}\right)^2\psi = \xi Y\left(\frac{\log_{10}10.6}{\log_{10}10.6\frac{X}{\Delta}}\right)^2 \frac{\rho_s-\rho_f}{\rho_f}\frac{D}{R'S_e} \tag{4}$$

where i_B = fraction of bed load in a given grain size

i_b = fraction of bed material in a given grain size

q_B = bed load rate in weight per unit of time and width

ρ_f, ρ_s = density of the fluid and solids respectively

D = grain size

R' = hydraulic radius with respect to the grain

S_e = energy gradient

Δ = the apparent roughness diameter

X = characteristic grain size of mixture

Y = pressure correction in transition smooth-rough

ξ = "hiding factor" of grains in a mixture

The equations 2 to 4 are derived from experiments with uniform sediment. They have such a form that they can equally well be applied to the individual grain sizes of mixtures. The correction factors ξ and Y are introduced only for non-uniform materials; their significance requires further explanation.

The factor, Y, is used to describe the change of the lift coefficient in a mixture, and is a function of K_s/δ (or, of the Reynolds number of the flow at the bed surface). The length K_s is the roughness diameter and δ is the thickness of laminar sublayer.

The factor, X, is defined as the largest particle size that will be subjected to the shielding effect by protruding coarser particles or by the laminar sublayer. The lift on the particles ($D < X$) must be corrected by division with a parameter ξ which is itself a function of D/X. The parameter, ξ, remains unity for D/X greater than one; after a short transition the curve $\xi = \xi$ (D/X) follows a straight line with a slope of 2 in a logarithmic graph. The value X was empirically found to be equal to 0.77Δ for a rough bed ($\Delta / \delta > 1.8$) and 1.39δ for a smooth bed ($\Delta / \delta < 1.8$). It is important to point out that the value X thus defined is over all average value for the given bed and flow, while Eq. 2 to 4 apply to individual grain sizes of the bed mixture. In detail, the largest particle size that will be subjected to the shielding effect actually varies from locality to locality on the bed. This is because a particle which is hidden behind a protruding large particle at the bed surface might lose this shielding effect if it happens to fall in another place where the surrounding particles are of smaller size. The same is true even for a smooth bed over which a laminar sublayer exists. There may still be a few coarse grains projecting above this sublayer and creating more protected zones behind them. The effective value of X in these regions is then larger than that in the rest of the bed.

The correction factor, ξ, is actually introduced to account for the consequence of the mutual interference between bed particles of different sizes, i.e., particles of a certain size in a mixture are not subjected to the same flow velocities as they are in the case where the entire bed is composed of material of its own size.

The original ξ curve is based on the results of a special set of 26 experiments conducted during the years 1944-46 at the California Institute of Technology by the senior author. Six sand mixtures were used with sorting coefficients varying from 0.74 to 0.88, with the exception of one which had a coefficient of 0.60. The small spread of the particle sizes in these mixtures makes the bed material move more or less as a unit. The segregation of the bed material is not a serious concern, which justifies the use of the original material as the bed material.

The question arises as to what will happen if the bed material and the transport include a wide range of sizes. Presumably the different intensities of motion of various particle sizes will result in some kind of segregation and rearrangement of bed surface. One of the fundamental assumptions used in deriving the bed load equation, namely, that particles are equally available at the surface and in the main body of the bed, may then no longer be valid. This in itself is a very important feature and calls for detailed study. With this in mind, the writers conducted a series of new experiments with well-graded materials at the University of California in 1950-51. The results of this study are presented in this paper.

During the course of these experiments, various methods of approach have been employed. In most of the runs, the particles moved over a bed which consisted of the particles despoited at the same flow. In a few of the runs, the conventional method of flume study was adopted, where the moving particles were eroded bed particles. The two methods of approach permit the experimental determination of a minimum and of a maximum equilibrium rate of bed load movement. Comparison of the results from these two methods of approach appears to be of basic interest. If these two methods give different results, the actual rate of sediment transport may assume any value between them without change of the bed, and may depend to some degree on the availability of the sediment from the watershed upstream.

The difference between the load and the minimum equilibrium rate actually represents what has been called wash load. These experiments thus provide an opportunity to have an insight into the concept of wash load. Questions like: "What is the basic difference between bed material load and wash load? Why does the bed material load develop a relationship between rate of transport and flow, while the rate of transport of wash load is a function only of its availability from the upstream watershed?" must find their respective answers in the results of this study.

Furthermore it will be seen that well-graded material has a tendency to deposit at high rates in the form of alternate coarse and fine layers. A review of the literature indicates that this is the first time that bed stratification is reported to be formed under constant flow conditions. This should prove very significant to geologists, who are interested in the history of river deposits.

It was also found that the rate of sediment motion, including both the bed material load and wash load, can be estimated for an aggrading river by applying the bed load function and a certain set of ξ curves if the bed composition and the flow at the time of deposition are known. The results thus obtained can be used to check the data of reservoir surveys which are carried out rather extensively at the present by the field engineers.